1/f NOISE CONVERSION IN ONE PORT MODEL OF CRYSTAL OSCILLATOR

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ABSTRACT

A single port 1/f model is examined of a crystal oscillator combined with a resonator and an equivalent negative resistance of amplifier. Intrinsic 1/f flicker fluctuations of resonator and amplifier amplitude and phase are converted to the oscillator amplitude and phase, and the proper transformation coefficients are applied. An analysis of the oscillator phase power spectral density is provided in detail for the particular case of a resonator excited in oscillator at various frequencies. Conformity with the Leeson's model is shown.

1. INTRODUCTION

Amplitude and phase spectral densities of crystal oscillator are shaped by the flicker and additive noises of both an amplifier and resonator. Leeson was the first who offered in 1966 [1] a physical but rather heuristic explanation of the forming mechanism. Then plenty of papers were devoted to the problem. An overwhelming majority of papers considers crystal resonator as just a source of 1/f noise with its different appearance within and beyond a bandwidth. Such an approach did not lead yet to appreciable results that is explained by the only reason: A resonator itself "hides" several intrinsic 1/f noise sources inside, which assemblage forms its amplitude and phase power spectral densities. First, we meet this hypothesis in the early work of Kuleshov and Janushevsky [2], the results then were accumulated in [3]. What is the essence? They just assumed the resonator motional inductance L_1, capacity C_1, resistance R_1, and even static capacity C_0 to be flicker noisy, have measured their flicker coefficients, and then analyzed their effect in the amplitude and phase either. If to recall that flicker noise has recently been reveled almost in each electronics unit [4] then such a hypothesis seems realistic. Employing this idea, we have analyzed in [5] a basic 1/f noise model of crystal resonator, provided the proper transformation coefficients for each 1/f source to the amplitude and phase, and examined the model for several experimental data published for over two decades by Wainright et al., Driscoll, Curtis, and Janushevsky. In each case the model fitted data more than well. Just to illustrate, we bring the Fig. 1, where the non trivial Curtis shape [5] was simulated almost precisely, except for the random frequency walk range below 100Hz.

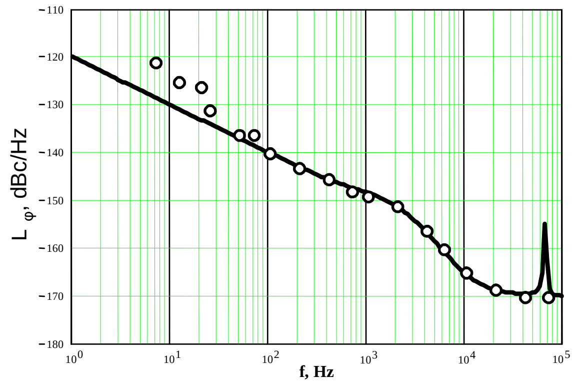

Figure 1. Measured (points, [6]) and simulated [5] phase power spectral density of the crystal resonator, 125 MHz

In this report, we consider a generalized one port oscillator model with a flicker noisy resonator and amplifier. We, first, convert 1/f intrinsic flicker fluctuations of a resonator and an amplifier to the oscillator amplitude and phase. Then we make analysis of the oscillator possible phase spectral densities. Finally, we show that, like a resonator case [5], the oscillator phase power spectral density may be efficiently predicted depending on weights of the particular noises. Conformity with the Leeson's model is noted.

2. ONE PORT NOISY MODEL OF CRYSTAL OSCILLATOR

Figure 2 shows a single-port model of crystal oscillator.

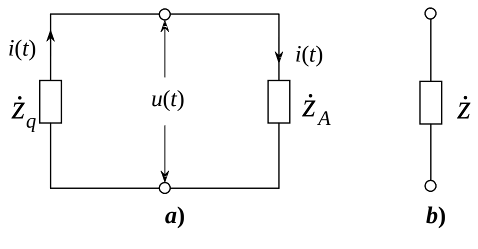

Figure 2. One port model of a crystal oscillator

Here $\dot{z}_q = Z_q e^{-j\varphi_q}$ is a complex impedance of crystal resonator with its noisy module Z_q and phase φ_q, $\dot{z}_A = Z_A e^{-j\varphi_A}$ is that of amplifier with its noisy module Z_A and phase φ_A. Transfer function of a resonator branch is $K_1(j\omega) = u/i = \dot{z}_q$, and that of an amplifier feedback in turn is $K_2(j\omega) = -i/u = -1/\dot{z}_A$. Instantly the steady state balance relation appears in a form of $K_1(j\omega)K_2(j\omega) = 1$ being satisfied by an equality

$$\dot{z}_q = -\dot{z}_A, \tag{1}$$

which produces the balance equations for the amplitude and phase, these are, respectively

$$Z_q = -Z_A, \tag{2}$$

$$\varphi_q - \varphi_A = 2\pi n,\ n = 0, 1,\ldots \tag{3}$$

Now assume either impedance of the model (Fig. 2) to be noisy, this is

$$\dot{z}_q = (\overline{Z}_q + \Delta Z_q)e^{j(\overline{\varphi}_q + \Delta\varphi_q)}, \tag{4}$$

$$\dot{z}_A = (\overline{Z}_A + \Delta Z_A)e^{j(\overline{\varphi}_A + \Delta\varphi_A)}, \tag{5}$$

where small amplitude fluctuations, $\Delta Z_q(t)$ and $\Delta Z_A(t)$, and small phase fluctuations, $\Delta\varphi_q(t)$ and $\Delta\varphi_A(t)$, are due to both additive thermal noises and flicker sources; $\overline{Z}_A$, $\overline{Z}_q$, and $\overline{\varphi}_q$, $\overline{\varphi}_A$ are noiseless (deterministic) modules and phases, respectively.

Oscillator amplitude and phase noise is associated with fluctuations of the module and phase, respectively, of its complex impedance. For the oscillator impedance (Fig. 2*b*) we then may use the same form, namely

$$\dot{z}(t) = (\overline{Z} + \Delta Z)e^{j(\overline{\psi} + \Delta\psi)}, \tag{6}$$

where $\overline{Z}$ and $\overline{\psi}$ are mean values, and $\Delta Z(t)$ and $\Delta\psi(t)$ are noisy adds to the oscillator amplitude and phase, respectively. Impedance (6) produces a conductivity

$$\frac{1}{\dot{z}} = \frac{1}{\dot{z}_q} + \frac{1}{\dot{z}_A}, \tag{7}$$

which with account of (4)—(6) transfers to

$$\frac{e^{-j(\overline{\psi}+\Delta\psi)}}{\overline{Z}\left(1 + \dfrac{\Delta Z}{\overline{Z}}\right)} = \frac{e^{-j\psi_1}}{\dfrac{\overline{Z}_q\overline{Z}_A}{Z_1}\left(1 + \dfrac{\overline{Z}_q\Delta Z_A + \overline{Z}_A\Delta Z_q}{\overline{Z}_q\overline{Z}_A}\right)}, \tag{8}$$

where an additional resistance is

$$Z_1 \cong \sqrt{\overline{Z}_A^2 + \overline{Z}_q^2 + 2\overline{Z}_q\overline{Z}_A\cos(\varphi_q - \varphi_A)}, \tag{9}$$

and a phase ψ_1 is written as

$$\psi_1 = -arctg\,\frac{\overline{x} + \Delta x}{\overline{y} + \Delta y}, \tag{10}$$

where the additional variables are readily expressed as

$$\overline{x} = \overline{Z}_q\sin\overline{\varphi}_A + \overline{Z}_A\sin\overline{\varphi}_q, \tag{11}$$

$$\overline{y} = \overline{Z}_A\cos\overline{\varphi}_q + \overline{Z}_q\cos\overline{\varphi}_A, \tag{12}$$

$$\Delta x = \overline{Z}_A\Delta\varphi_q\cos\overline{\varphi}_q + \Delta Z_A\sin\overline{\varphi}_q + \overline{Z}_q\Delta\varphi_A\cos\overline{\varphi}_A + \Delta Z_q\sin\overline{\varphi}_A, \tag{13}$$

$$\Delta y = -\overline{Z}_A\Delta\varphi_q\sin\overline{\varphi}_q + \Delta Z_A\cos\overline{\varphi}_q - \overline{Z}_q\Delta\varphi_A\sin\overline{\varphi}_A + \Delta Z_q\cos\overline{\varphi}_A. \tag{14}$$

It now follows straightforward from the equality of the second terms in the denominators of (8) that an oscillator amplitude fractional noise $\delta_Z = \Delta Z/\overline{R}_1$ is formed by an additive sum of the weighted fractional amplitude fluctuations of a resonator and an amplifier, $\delta_q = \Delta Z_q/\overline{R}_1$ and $\delta_A = \Delta Z_A/\overline{R}_1$, via the relation

$$\delta_Z = K_{ZqZ}\delta_q + K_{ZAZ}\delta_A, \tag{15}$$

where the transformation coefficients are given by

$$K_{ZqZ}(\mathrm{f}) = \frac{k_q(\mathrm{f})}{\sqrt{k_A^2 + k_q^2(\mathrm{f}) + 2k_A k_q(\mathrm{f})\cos[\varphi_q(\mathrm{f}) - \varphi_A]}}, \tag{16}$$

$$K_{ZAZ}(\mathrm{f}) = \frac{k_A}{\sqrt{k_A^2 + k_q^2(\mathrm{f}) + 2k_A k_q(\mathrm{f})\cos[\varphi_q(\mathrm{f}) - \varphi_A]}}, \tag{17}$$

$k_A = \overline{Z}_A/\overline{R}_1$, $k_q(\mathrm{f}) = \overline{Z}_q(\mathrm{f})/\overline{R}_1$, f is Fourier frequency, and we tacitly assumed that the amplifier phase seems constant relatively the resonator phase response change.

This is not, however, the case for the oscillator phase noise. To find the proper formula, we need to calculate a difference between a total noisy phase and its average (deterministic) trend. Based on (10), it becomes

$$\Delta\psi = X - Y = -\mathrm{arctg}\frac{\overline{x} + \Delta x}{\overline{y} + \Delta y} + \mathrm{arctg}\frac{\overline{x}}{\overline{y}}. \tag{18}$$

Observing that $-1 < XY$ and reminding that $\Delta\psi$ is inherently small transfers (18) to

$$\Delta\psi \cong \frac{\overline{x}\Delta y - \overline{y}\Delta x}{\overline{y}(\overline{y} + \Delta y) + \overline{x}(\overline{x} + \Delta x)}. \tag{19}$$

Now substitute (11)—(14) for (19), provide routine transform, and finally come to the generalized form

$$\Delta\psi \cong K_{Zq\psi}\delta_q + K_{\varphi q\psi}\Delta\varphi_q + K_{ZA\psi}\delta_A + K_{\varphi A\psi}\Delta\varphi_A, \tag{20}$$

where the transformation coefficients of the resonator and amplifier amplitude and phase fluctuations, δ_q and $\Delta\varphi_q$, δ_A and $\Delta\varphi_A$, are given, respectively, by

$$K_{Zq\psi}(f) = \frac{k_A}{k^2}\sin[\overline{\varphi}_q(f) - \overline{\varphi}_A], \tag{21}$$

$$K_{\varphi q\psi}(f) = -\frac{k_A}{k^2}\{k_q(f)\cos[\overline{\varphi}_q(f) - \overline{\varphi}_A] + k_A\}, \tag{22}$$

$$K_{ZA\psi}(f) = -\frac{k_q(f)}{k^2}\sin[\overline{\varphi}_q(f) - \overline{\varphi}_A]\,, \qquad (23)$$

$$K_{\varphi A\psi}(f) = -\frac{k_q(f)}{k^2}\{k_A\cos[\overline{\varphi}_q(f) - \overline{\varphi}_A] + k_q(f)\}\,, \quad (24)$$

where $k^2 = k_A^2 + k_q^2(f) + 2k_A k_q(f)\cos[\overline{\varphi}_q(f) - \overline{\varphi}_A]$.

3. OSCILLATOR PHASE POWER SPECTRAL DENSITY

Intrinsic fluctuations of resonator and amplifier are uncorrelated. Let us assume their phase and amplitude wanders to be uncorrelated[1] and stationary. Then calculate correlation functions of the left and right parts of (20), apply the Wiener-Khinchin theorem and transfer to the one-sided power spectral densities,

$$\begin{aligned}\mathrm{S}_\psi(\mathrm{f}) \cong\, & K_{Zq\psi}^2(\mathrm{f})\mathrm{S}_{Zq}(\mathrm{f}) + K_{\varphi q\psi}^2(\mathrm{f})\mathrm{S}_{\varphi q}(\mathrm{f}) \\ & + K_{ZA\psi}^2(\mathrm{f})\mathrm{S}_{ZA}(\mathrm{f}) + K_{\varphi A\psi}^2(\mathrm{f})\mathrm{S}_{\varphi A}(\mathrm{f})\,, \quad (25)\end{aligned}$$

where a resonator amplitude noise spectral density for the uncorrelated intrinsic fluctuations is given by

$$\begin{aligned}\mathrm{S}_{Zq}(\mathrm{f}) \cong \Bigg[& \beta_R \frac{K_{RZ}^2(\mathrm{f}) + K_{RZ}^2(-\mathrm{f})}{2} \\ & + (\beta_L + \beta_C)\frac{K_{XZ}^2(\mathrm{f}) + K_{XZ}^2(-\mathrm{f})}{2} \\ & + \beta_{C0}\frac{K_{0Z}^2(\mathrm{f}) + K_{0Z}^2(-\mathrm{f})}{2}\Bigg]\mathrm{f}^{-1}\,, \quad (26)\end{aligned}$$

where all the transformation coefficients are obtained in [5]. A resonator phase noise spectral density has almost the same form of (26) but simplifies to, making for the numerical values of the flicker-noise coefficients [3],

$$\begin{aligned}\mathrm{S}_{\varphi q}(\mathrm{f}) \cong \Bigg[& \beta_R \frac{K_{R\varphi}^2(\mathrm{f}) + K_{R\varphi}^2(-\mathrm{f})}{2} \\ & + (\beta_L + \beta_C)\frac{K_{X\varphi}^2(\mathrm{f}) + K_{X\varphi}^2(-\mathrm{f})}{2}\Bigg]\mathrm{f}^{-1}\,. \quad (27)\end{aligned}$$

Based on [7] and [8], the amplifier one-sided phase noise spectral density may be modeled by

$$\mathrm{S}_{\varphi A}(\mathrm{f}) = \beta_\varphi \mathrm{f}^{-1} + \mathrm{S}_{\varphi NF}\,, \qquad (28)$$

where $\mathrm{S}_{\varphi NF}$ is a phase noise floor. Likewise, the one-sided amplitude noise spectral density in average exhibits the magnitude higher by 4-5 dBc at f = 1Hz and by 2-4 dBc at 1kHz being changed by the same law

$$\mathrm{S}_{ZA}(\mathrm{f}) = \beta_A \mathrm{f}^{-1} + \mathrm{S}_{ANF}\,, \qquad (29)$$

[1] We use this, however, without a proper proof

where a noise floor S_{ANF} is due to a Johnson noise, $S_{ANF} = 2kT/P_A$, where P_A is an amplifier dissipative power. Substituting (26)—(29) for (25) completes this formula, which we will investigate here numerically.

4. NUMERICAL STUDIES

In this section we simulate the crystal oscillator phase noise spectral density (25) for the resonator (Fig. 1) and an amplifier with given amplitude and phase noises.

Given a crystal resonator with the parameters taken from [5] and [6]: $f_s = 125$ MHz, $Q = 3.13\cdot 10^4$, $C_0 = 1.06$ pF, $R_1 = 36$ Ohm, $\kappa \cong 0.03$, $\beta_R = 5\cdot 10^{-12}$, $\beta_L = \beta_C = 5.1\cdot 10^{-22}$, and $\beta_{C0} = 5\cdot 10^{-14}$. The feedback amplifier phase noise is simulated with $\beta_\varphi = 3.162\cdot 10^{-13}$ and $\mathrm{S}_{\varphi NF} = 3.162\cdot 10^{-15}$, this is -120 dBc/Hz at 1 Hz and -141 dBc/Hz at 100 kHz. The amplifier amplitude noise is, respectively, with $\beta_A = 1\cdot 10^{-12}$, $\mathrm{S}_{ANF} = 7.943\cdot 10^{-15}$, -115 dBc/Hz at 1 Hz, and -137 dBc/Hz at 100 kHz.

Figure 3 illustrates the case of the excitation angle 45º (Colpitts mode) exhibiting the spectral density shapes if to account step by step all the internal sources, namely: the resonator phase noise (**A**), added the resonator amplitude noise (**B**), added the amplifier amplitude noise (**C**), and, finally, added the amplifier phase noise (**D**). To compare, the curves are given along with the resonator intrinsic (unloaded) phase noise.

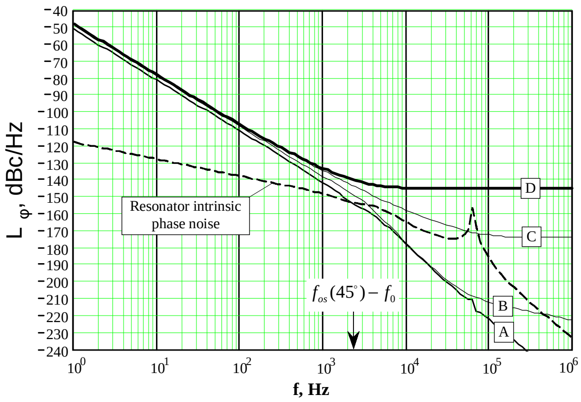

Figure 3. Phase spectral density for the Colpitts mode

Assuming the only source acting in oscillator, this is a resonator phase noise, the spectral density occupies the lowest possible position (**A**). Furthermore, it turns out that the resonator intrinsic noise is just gained left the frequency $\mathrm{f} = f_{os}(45^\circ) - f_0$ for the slope f^{-3}, and attenuated right it, where $f_0 = 1/2\pi\sqrt{L_1C_1}$. It is however an isolated case so long as accounting of the resonator intrinsic amplitude noise (**B**) affects the shape, and then the curve (**B**) may truly be treated as an oscillator limited noise with a theoretically noiseless amplifier. Herewith adding the amplifier amplitude noise updates the spectral density only beyond a bandwidth (**C**). Finally, accounting the amplifier phase noise elevates the phase spectral density to its actual shape (**D**), this in fact looks like a typically measured. We then conclude for this case that *within a bandwidth the oscillator phase noise is mainly due to a resonator phase noise* and that *beyond it to the amplifier phase noise*.

We may now compare the rigorous enough formula (25) to the Leeson's model. Basically, to provide, we must just approximate the curve (25) by the extended Leeson polynomial

$$S_L(f) = a_3 f^{-3} + a_2 f^{-2} + a_1 f^{-1} + a_0 \qquad (30)$$

and answer on whether it fits (25) well. As a matter of fact we note that with $a_0 = 2.967 \cdot 10^{-15}$, $a_1 = 0$, $a_3 = 1.506 \cdot 10^{-5}$, and $a_2 = 4.802 \cdot 10^{-8}$ the purely Leeson model ($a_1 = 0$) trends within a simulated density almost precisely. Figure 4 illustrates the result for the Colpitts oscillator (45º), and we may think both models are consistent.

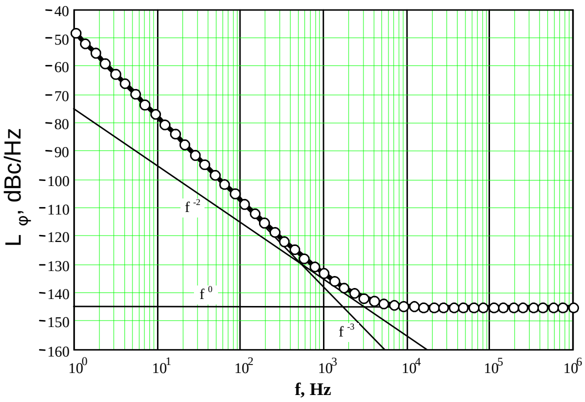

Figure 4. Conformity of the model (25) (circles) and the Leeson's model (bold curve) for the case of 45º

7. CONCLUSIONS

The most important conclusions for the considered example follow straightforward from Fig. 3, namely:

- *Within a bandwidth*, the oscillator phase noise *in the Colpitts oscillator* is mostly due to a *resonator phase noise*.
- Otherwise, *beyond a bandwidth*, the oscillator phase noise mostly "thanks" the *amplifier phase noise*.

We have presented the new oscillator phase noise model and examined it for the particular case of crystal resonator, 125 MHz. Certainly this only case constrains us to be careful with generalization. Nevertheless, we would like to note:

- The model (25) is consistent with the Lesson's model (30), so that it seems the coefficients of (30) may be analytically determined with a reasonable accuracy. It however does not exclude that the extended model (30) fits better in some other cases.
- The model (25) allows calculation the oscillator phase noise spectral density for arbitrary shapes of resonator and amplifier intrinsic amplitude and phase spectral densities.
- It follows from the considered example that within a resonator bandwidth the resonator noise dominants whereas beyond it the amplifier noise is more active.

8. REFERENCES

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